中原大學 113 學年度第 2 學期 微積分 A 群 第二次會考

I. 填充題. (25 分)

1. Find $\lim_{(x,y)\to(\pi,\pi/2)} y\sin(x-y) =$ $\underline{\quad \pi/2 \quad}$

2. If $w=\ln(3x+2y+z)$, then $\dfrac{\partial w}{\partial x} =$ $\underline{\quad \dfrac{3}{3x+2y+z} \quad}$

3. If $z=\sin x\cos y, x=\sqrt{t}, y=1/t$, then $\dfrac{dz}{dt} =$ $\underline{\quad \dfrac{\cos\sqrt{t}\cdot\cos\frac{1}{t}}{2\sqrt{t}} + \dfrac{\sin\sqrt{t}\cdot\sin\frac{1}{t}}{t^2} \quad}$

4. If $f(x,y)=x^2+2xy-3y^2$ and $\mathbf{u}=\dfrac{1}{2}\mathbf{i}+\dfrac{\sqrt{3}}{2}\mathbf{j}$, then $D_{\mathbf{u}}f(2,1) =$ $\underline{\quad 3-\sqrt{3} \quad}$

5. If $f(x,y)=x/y$, then $\nabla f(1,2) =$ $\underline{\quad \left(\dfrac{1}{2},\dfrac{-1}{4}\right) \text{ or } \left[\dfrac{1}{2},\dfrac{-1}{4}\right]^T \text{ or } \dfrac{1}{2}\mathbf{i}+\dfrac{-1}{4}\mathbf{j} \quad}$

II. 計算、證明題. (80 分)

1. Let $f(x,y) = x^2 - 2xy + y^2$. Using the limit definition of partial derivative to show $f_x(x,y) = 2x - 2y$.

2. Show that the limit does not exist: (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$. (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2\sin^2 x}{x^4+y^4}$.

3. Find (a) $\displaystyle\lim_{(x,y)\to(0,0)} xy\sin\frac{1}{x^2+y^2}$. (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$.

4. Find the first partial derivatives of the function: (a) $f(x,y) = x^3\sin y$. (b) $f(x,y) = \displaystyle\int_y^x \sin(e^t)\,dt$.

5. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$: (a) $x^2 + 2y^2 + 3z^2 = 1$. (b) $yz + x\ln y = z^2$.

6. Find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$: (a) $z = (x-y)^5, x = st^2, y = s^2t$. (b) $z = \ln(2x+3y), x = s\cos t$, $y = t\sin s$.

7. Find the directional derivative of f at the given point P in the direction of the vector $\mathbf{u}$:
(a) $f(x,y) = x^2e^y, P(3,1), \mathbf{u} = -5\mathbf{i}+12\mathbf{j}$. (b) $f(x,y,z) = x^2yz - xyz^3, P(2,-1,1)$, $\mathbf{u} = 4\mathbf{j} - 3\mathbf{k}$.

8. Find $\nabla f(P)$: (a) $f(x,y) = \sin(xy), P(1,0)$. (b) $f(x,y,z) = \dfrac{x}{y+z}, P(8,1,3)$.

9. Let $f(x,y,z) = x\ln(yz)$ and let $\mathbf{u}^*$ be the unit vector that maximizes the directional derivative $D_{\mathbf{u}}f(1,2,\frac{1}{2})$. Find $\mathbf{u}^*$ and $D_{\mathbf{u}^*}f(1,2,\frac{1}{2})$.

10. Consider the equation $x+y+z = e^{xyz}$. Find the tangent plane and the normal line at $P(0,0,1)$.

113 學年度第 2 學期理、工、電資學院微積分 (A 群) 期中考答案 2025.5.14

題號	答案	來源
1	略	14.3 – 定義應用
2	略	14.2 – 習題 13,17
3	$(a)0, \quad (b)0$	14.2 – 習題 31,51
4	$(a)f_x(x,y)=3x^2\sin y, f_y(x,y)=x^3\cos y,$ $(b)f_x(x,y)=\sin(e^x), f_y(x,y)=-\sin(e^y)$	14.3 – 習題 11,25
5	$(a)z_x=\frac{-x}{3z}, z_y=\frac{-2y}{3z}. \quad (b)z_x=\frac{-\ln y}{y-2z}, z_y=\frac{-z-\frac{x}{y}}{y-2z}$	14.3 – 習題 41,44
6	$(a)\frac{\partial z}{\partial s}=5(st^2-s^2t)^4(t^2-2st), \frac{\partial z}{\partial t}=5(st^2-s^2t)^4(2st-s^2).$ $(b)\frac{\partial z}{\partial s}=\frac{2\cos t+3t\cos s}{2s\cos t+3t\sin s}.\frac{\partial z}{\partial t}=\frac{-2s\sin t+3\sin s}{2s\cos t+3t\sin s}$	14.5 – 習題 11,13
7	$(a)6e, \quad (b)\frac{2}{5}$	14.6 – 習題 11
8	$(a)\nabla f(1,0)=(0,1), \quad (b)\nabla f(8,1,3)=\left(\frac{1}{4},\frac{-1}{2},\frac{-1}{2}\right)$	14.6 – 習題 29,31
9	$D_{\mathbf{u}^*}f\left(1,2,\frac{1}{2}\right)=\frac{\sqrt{17}}{2}, \mathbf{u}^*=\left(0,\frac{\sqrt{17}}{17},\frac{4\sqrt{17}}{17}\right)$	14.6 – 習題 30
10	The equation of the tangent plane is $x+y+z-1=0,$ The normal line are $\frac{x-0}{1}=\frac{y-0}{1}=\frac{z-1}{1}$	14.6 – 習題 51

* 為非勾選習題、例題或勾選習題類似題.

證明題、做圖題過程略過.