

I. 填充題. (25 分)

1. Find $\lim_{(x,y) \rightarrow (\pi, \pi/2)} y \sin(x - y) = \underline{\pi/2}$

2. If $w = \ln(3x + 2y + z)$, then $\frac{\partial w}{\partial x} = \underline{\frac{3}{3x + 2y + z}}$

3. If $z = \sin x \cos y$, $x = \sqrt{t}$, $y = 1/t$, then $\frac{dz}{dt} = \underline{\frac{\cos \sqrt{t} \cdot \cos \frac{1}{t}}{2\sqrt{t}} + \frac{\sin \sqrt{t} \cdot \sin \frac{1}{t}}{t^2}}$

4. If $f(x, y) = x^2 + 2xy - 3y^2$ and $\mathbf{u} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$, then $D_{\mathbf{u}}f(2, 1) = \underline{3 - \sqrt{3}}$

5. If $f(x, y) = x/y$, then $\nabla f(1, 2) = \underline{\left(\frac{1}{2}, \frac{-1}{4}\right) \text{ or } \left[\frac{1}{2}, \frac{-1}{4}\right]^T \text{ or } \frac{1}{2}\mathbf{i} + \frac{-1}{4}\mathbf{j}}$

II. 計算、證明題. (80 分)

1. Let $f(x, y) = x^2 - 2xy + y^2$. Using the limit definition of partial derivative to show $f_x(x, y) = 2x - 2y$.
2. Show that the limit does not exist: (a) $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2}{x^2 + y^2}$. (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$.
3. Find (a) $\lim_{(x,y) \rightarrow (0,0)} xy \sin \frac{1}{x^2 + y^2}$. (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + y^3}{x^2 + y^2}$.
4. Find the first partial derivatives of the function: (a) $f(x, y) = x^3 \sin y$. (b) $f(x, y) = \int_y^x \sin(e^t) dt$.
5. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$: (a) $x^2 + 2y^2 + 3z^2 = 1$. (b) $yz + x \ln y = z^2$.
6. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$: (a) $z = (x-y)^5, x = st^2, y = s^2t$. (b) $z = \ln(2x+3y), x = s \cos t, y = t \sin s$.
7. Find the directional derivative of f at the given point P in the direction of the vector $\mathbf{u}$:
(a) $f(x, y) = x^2 e^y, P(3, 1), \mathbf{u} = -5\mathbf{i} + 12\mathbf{j}$. (b) $f(x, y, z) = x^2 yz - xyz^3, P(2, -1, 1), \mathbf{u} = 4\mathbf{j} - 3\mathbf{k}$.
8. Find $\nabla f(P)$: (a) $f(x, y) = \sin(xy), P(1, 0)$. (b) $f(x, y, z) = \frac{x}{y+z}, P(8, 1, 3)$.
9. Let $f(x, y, z) = x \ln(yz)$ and let $\mathbf{u}^*$ be the unit vector that maximizes the directional derivative $D_{\mathbf{u}} f(1, 2, \frac{1}{2})$. Find $\mathbf{u}^*$ and $D_{\mathbf{u}^*} f(1, 2, \frac{1}{2})$.
10. Consider the equation $x + y + z = e^{xyz}$. Find the tangent plane and the normal line at $P(0, 0, 1)$.

題號	答案	來源
1	略	14.3 – 定義應用
2	略	14.2 – 習題 13,17
3	(a)0, (b)0	14.2 – 習題 31,51
4	(a) $f_x(x, y) = 3x^2 \sin y, f_y(x, y) = x^3 \cos y,$ (b) $f_x(x, y) = \sin(e^x), f_y(x, y) = -\sin(e^y)$	14.3 – 習題 11,25
5	(a) $z_x = \frac{-x}{3z}, z_y = \frac{-2y}{3z}.$ (b) $z_x = \frac{-\ln y}{y-2z}, z_y = \frac{-z - \frac{x}{y}}{y-2z}$	14.3 – 習題 41,44
6	(a) $\frac{\partial z}{\partial s} = 5(st^2 - s^2t)^4(t^2 - 2st), \frac{\partial z}{\partial t} = 5(st^2 - s^2t)^4(2st - s^2).$ (b) $\frac{\partial z}{\partial s} = \frac{2 \cos t + 3t \cos s}{2s \cos t + 3t \sin s} \cdot \frac{\partial z}{\partial t} = \frac{-2s \sin t + 3 \sin s}{2s \cos t + 3t \sin s}$	14.5 – 習題 11,13
7	(a) $6e,$ (b) $\frac{2}{5}$	14.6 – 習題 11
8	(a) $\nabla f(1, 0) = (0, 1),$ (b) $\nabla f(8, 1, 3) = \left(\frac{1}{4}, \frac{-1}{2}, \frac{-1}{2}\right)$	14.6 – 習題 29,31
9	$D_{\mathbf{u}^*} f\left(1, 2, \frac{1}{2}\right) = \frac{\sqrt{17}}{2}, \mathbf{u}^* = \left(0, \frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17}\right)$	14.6 – 習題 30
10	The equation of the tangent plane is $x + y + z - 1,$ The normal line are $\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{1}$	14.6 – 習題 51

* 為非勾選習題、勾選習題類似題.

證明題、做圖題過程略過.