

科目名稱: 微積分 (下)(A 群) 開課班級: A 群微積分 考試時間: 6 月 18 日 第二節

I. 填充題. (25 分)

1. The critical points of $f(x, y) = 2x^2 - 8xy + y^4 + 4y^3$ are $(0, 0)$, $(2, 1)$, and $(-8, -4)$
2. If $R = \{(x, y) | -1 \leq x \leq 1, -1 \leq y \leq 1\}$, then $\iint_R \sqrt{1-x^2} dA =$ π
3. If $R = [0, \pi] \times [0, \pi]$, then $\iint_R \sin(x^2) \cos y dA =$ 0
4. The volume of the solid that under the paraboloid $z = x^2 + y^2$ and above the disk $x^2 + y^2 \leq 4$ is 8π
5. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n} (2n)!} = 1 - \frac{(\pi/3)^2}{2!} + \frac{(\pi/3)^4}{4!} - \frac{(\pi/3)^6}{6!} + \dots =$ $\frac{1}{2}$

II. 計算、證明題. (80 分)

1. Find the critical points of $f(x, y) = 4 + x^3 + y^3 - 3xy$. Use the second derivative test to describe the behavior of f (local maximum, local minimum or saddle point) at these critical points.
2. The function $f(x, y) = x^2 + y^2 + x^2y + 4$ has three critical points $(0, 0)$, $(\sqrt{2}, 1)$ and $(-\sqrt{2}, 1)$. Find the maximum and minimum values of f on the set $D = \{(x, y) \mid |x| \leq 1, |y| \leq 1\}$.
3. (a) Find $\int_0^2 y\sqrt{x+2} dx$ and $\int_0^3 y\sqrt{x+2} dy$. (b) Evaluate $\int_{-3}^3 \int_0^{\frac{\pi}{2}} (y + y^2 \cos x) dx dy$.
4. (a) Evaluate $\int_0^1 \int_0^y x e^{y^3} dx dy$. (b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\cos x} \sin^2 x dy dx$.
5. Evaluate $\int_0^1 \int_{\arcsin y}^{\frac{\pi}{2}} \cos x \sqrt{1 + \cos^2 x} dx dy$ by reversing the order of integration.
6. Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} dy dx$ by converting to polar coordinates.
7. Find the area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$.
8. Use the polar coordinates to find the volume of the solid bounded by the paraboloids $z = 6 - x^2 - y^2$ and $z = 2x^2 + 2y^2$.
9. (a) Use the definition to find the n th Maclaurin polynomial $P_n(x)$ of the function $f(x) = e^x$.
(b) Show that the Maclaurin series of $\ln \left(\frac{1+x}{1-x} \right)$ is $2 \left(\frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \right)$.
10. Use the Maclaurin series to find $f^{(203)}(0)$, where $f(x) = x \sin(x^2)$.

題號	答案	來源
1	$(0, 0), (1, 1)$ are critical points. $(0, 0)$ is a saddle point. $f(1, 1)$ is a local minimum.	14.7 – 習題 3*
2	the maximum value of f is 7, the minimum value of f is 4.	14.7 – 習題 35
3	(a) $\frac{(16 - 4\sqrt{2})y}{3}$ and $\frac{9}{2}\sqrt{x+2}$, (b) 18	15.1 – 習題 14, 19
4	(a) $\frac{1}{6}(e-1)$, (b) $\frac{1}{3}$	15.2 – 習題 3, 21
5	$\frac{1}{3}(2\sqrt{2}-1)$	15.2 – 習題 65
6	$\frac{\pi}{4}(1 - e^{-4})$	15.3 – 習題 39
7	$\frac{\pi}{8}$	15.3 – 例題 4
8	6π	15.3 – 習題 36
9	(a) 略, (b) 略	補充講義
10	$f^{(203)}(0) = \frac{203!}{101!}$	11.10 – 習題 93

* 為非勾選習題、例題或勾選習題類似題。
證明題、做圖題過程略過。