中原大學 113 學年度 □上學期 ■下學期 考試命題紙 ■期末考

科目名稱: 微積分 (下)(B 群)　　開課班級:B 群微積分　　考試時間: 6 月 18 日第二節

I. 填充題. (25 分)

1. Find $\lim_{(x,y)\to(1,1)} \dfrac{x^2y+xy}{5xy^2+y} = \underline{\ \dfrac{1}{3}\ }$

2. The domain of $g(x,y)=\sqrt{1-x^2-y^2}$ is $\underline{\ \{(x,y)\in R^2 | x^2+y^2\leq 1\}\ }$

3. If $f(x,y)=x^2y+y^2+\ln y$, then $\dfrac{\partial f}{\partial x} = \underline{\ 2xy\ }$

4. The third term (第三項) of the Maclaurin series representation for $f(x)=\dfrac{x^2+3x+5}{x-1}$ is $\underline{\ -9x^2\ }$

5. If $f(x,y)=x^2y^3+\dfrac{\sin y^2+1}{1+y^2}$, then $f_{yx} = \underline{\ 6xy^2\ }$

II. 計算、證明題. (80 分)

1. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ does not exist.

2. Show that $\displaystyle\lim_{(x,y)\to(0,1)} \frac{x^2y-x^2}{x^2+y^2-2y+1}$ exist.

3. Find and sketch the domain for $f(x,y)=\dfrac{2x}{y-x^2}$.

4. Sketch the graph of $f(x,y)=\sqrt{x^2+y^2}$. (只要畫出圖形)

5. Let $f(x,y)=\sin(x^2y)+x^3+y^2+8$. Find f_{xy}, f_{yx}, f_{xx} and f_{yy}.

6. Let $f(x,y)=\displaystyle\int_x^y \sin t^2\,dt$. Find $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$.

7. Find the Taylor polynomial $P_3(x)$ of degree 3 for $f(x)=xe^x$ at $c=1$.

8. Find the Maclaurin polynomial $P_4(x)$ of degree 4 for $f(x)=x^2\sin x$.

9. Let $f(x,y)=\begin{cases} \dfrac{x^4}{x^2+y^2} & \text{, if } (x,y)\neq(0,0) \\ 1 & \text{, if } (x,y)=(0,0) \end{cases}$. Show that $f(x,y)$ is not continuous at $(0,0)$.

10. Find $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2y^2z^2}{x^2+y^2+z^2}$.

113 學年度第 2 學期理、工、電資學院微積分 (B 群) 期末考答案 2025.6.18

題號	答案	來源
1	略	12.2 – 例題 2.5
2	略	12.2 – 習題 31
3	略	12.1 – 例題 1.1
4	略	12.1 – 例題 1.3*
5	$f_{xx} = -4x^2y^2\sin(x^2y) + 2y\cos(x^2y) + 6x$ $f_{xy} = -2x^3y\sin(x^2y) + 2x\cos(x^2y)$ $f_{yy} = -x^4\sin(x^2y) + 2$ $f_{yx} = -2x^3y\sin(x^2y) + 2x\cos(x^2y)$	12.3 – 例題 3.4
6	$\dfrac{\partial f}{\partial x} = -\sin x^2, \dfrac{\partial f}{\partial y} = \sin y^2$	12.3 – 習題 7
7	$P_3(x) = e = 2e(x-1) + \dfrac{3e}{2}(x-1)^2 + \dfrac{2e}{3}(x-1)^3$	補充講義
8	$P_4(x) = x^3$	補充講義
9	略	12.2 – 例題 2.8*
10	$\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2y^2z^2}{x^2+y^2+z^2} = 0$	12.2 – 習題 34*

* 為非勾選習題、例題或勾選習題類似題.
證明題、做圖題略過.