中原大學 114 學年度 ■上學期 □下學期 考試命題紙■期中考

科目名稱: 微積分 (上)(B 群)
考試時間: 11 月 5 日第二節

I. 填充題. (25 分)

1. The tangent line to $y = x^3 + x + 1$ at $x = 1$ is $\underline{\ y = 4x - 1\ }$

2. If $f(x) = \sqrt[3]{x} + \dfrac{1}{\sqrt{x}}$ then $f'(x) = \underline{\ \dfrac{1}{3}x^{-2/3} - \dfrac{1}{2}x^{-3/2}\ }$

3. The derivative of $f(x) = (x+1)^3$ is $f'(x) = \underline{\ 3(x+1)^2\ }$

4. If $f(x) = \dfrac{4x^2 + x + 1}{\sqrt{x}}$ then $f'(x) = \underline{\ \dfrac{12x^2 + x - 1}{2x\sqrt{x}}\ }$

5. If $f'(x)$ exists and $g(x) = (f(x))^2$ then $g'(x) = \underline{\ 2f(x)f'(x)\ }$
(用 $f(x)$ 與 $f'(x)$ 表示答案)

II. 計算、證明題. (80 分)

1. Find an equation of the tangent line to $y = \dfrac{2}{x+1}$ at $x = 2$.

2. Use the definition of derivative to show that if $f(x) = \sqrt[3]{x}$ then $f'(x) = \dfrac{1}{3}x^{-\frac{2}{3}}$.

3. Use the definition of derivative to show that if $f(x) = \dfrac{1}{\sqrt[3]{x}}, x \neq 0$ then $f'(x) = -\dfrac{1}{3}x^{-\frac{4}{3}}$.

4. If $f(x)$ is differentiable at $x = a$, show that $f(x)$ is continuous at $x = a$.

5. Use the product rule of derivative to find $f'(x)$ if $f(x) = \sqrt{x^2 + \sqrt{x}}$.
(不使用導數的乘法公式計算不予計分)

6. Show that $f(x) = \begin{cases} 4+x & ,x < 2 \\ 3x & ,x \geq 2 \end{cases}$ is not differentiable at $x = 2$.

7. Show that $f(x) = |x|$ is not differentiable at $x = 0$.

8. Find an equation of the tangent line to $f(x) = (x^3 + x + 1)(3x^2 + 2x - 1)$ at $x = 1$.

9. Use the definition of derivative to show that $f(x) = \begin{cases} x^3 \sin \frac{1}{x} & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$ is differentiable at $x = 0$.

10. If $f(x) = \left[\left(1 + \dfrac{1}{x}\right)^{-1} + 5\right]^{-1}$, use the quotient rule of derivative to find $f'(x)$.

114 學年度第 1 學期理、工、電資學院微積分 (B 群) 期中考答案 2025.11.5

題號	答案	來源
1	The tangent line is $y-\frac{2}{3}=-\frac{2}{9}(x-2)$	2.1 – 例題 1*
2	證明題	2.2 – 例題 4*
3	證明題	2.2 – 例題 3*
4	證明題	2.2 – 例題 1
5	$f'(x)=\frac{2x+\frac{1}{2\sqrt{x}}}{2\sqrt{x^2+\sqrt{x}}}$	2.4 – Product rule 概念題
6	證明題	2.2 – 例題 7*
7	證明題	2.2 – 定理
8	The tangent line is $y-12=40(x-1)$	2.4 – 習題 18
9	證明題	2.2 – 例題 7*
10	$f'(x)=\frac{-1}{(1+1/x)^2[(1+1/x)^{-1}+5]^2x^2}$	2.4 – Quotient rule 概念題

* 為非勾選習題、勾選習題類似題.
證明題過程略過.