

科目名稱: 微積分 (上)(A 群)

考試時間: 10 月 1 日 第二節

I. 填充題. (25 分)

1. The vertical asymptote of curve $y = \frac{x^2}{x-1}$ is $x = 1$

2. Let $f(x) = \begin{cases} 1-x, & x < 1 \\ \frac{1}{x}, & x \geq 1 \end{cases}$, find $\lim_{x \rightarrow 1} f(x) = \underline{\text{Does not exist}}$

3. The greatest integer function is defined by $[x]$ = the largest integer that is less than or equal to x .

Then $\lim_{x \rightarrow 3.5} [x] = \underline{3}$

4. Let $f(x) = \sqrt{x} + \frac{1}{x-1} + \frac{x}{x-2}$. On what intervals is $f(x)$ continuous? Ans : $\underline{[0, 1), (1, 2), (2, \infty)}$

5. Suppose that $f(3) = 1, g(3) = 2, f'(3) = 3$, and $g'(3) = 4$. Then $\left(\frac{f}{g}\right)'(3) = \underline{\frac{1}{2}}$

II. 計算、證明題. (80 分)

1. For the function whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \rightarrow 1} f(x)$ (b) $\lim_{x \rightarrow 3^-} f(x)$ (c) $\lim_{x \rightarrow 3^+} f(x)$ (d) $\lim_{x \rightarrow 3} f(x)$ (e) $f(3)$



2. Prove that $\lim_{x \rightarrow 5} \frac{|x - 5|}{x - 5}$ does not exist.

3. Evaluate (a) $\lim_{x \rightarrow 2} \frac{2 - x}{\sqrt{x + 2} - 2}$. (b) $\lim_{x \rightarrow -5} \frac{2x^2 + 9x - 5}{x^2 - 25}$ (不可使用羅必達).

4. Prove that $\lim_{x \rightarrow 0} x^4 \cos \frac{2}{x} = 0$.

5. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & , x < 2 \\ x^3 - cx & , x \geq 2 \end{cases}.$$

6. Show that there is a solution of the equation $x^3 - 4x - 1 = 0$ between -1 and 0 .

7. Let $f(x) = |x|$. (a) Prove that $f(x)$ is continuous at 0 . (b) Prove that $f(x)$ is not differentiable at 0 .

8. Find the derivative of the following function using the definition of derivative $f(x) = 4 + 3x - 7x^2$.

9. Find equations of tangent line and normal line to the curve $y = \frac{x}{1 + x^2}$ at point $(1, \frac{1}{2})$.

10. Find the derivatives of the following function (a) $f(x) = (4x^2 + 3)(2x + 5)$. (b) $f(x) = \frac{x^2 + x + 1}{x^2 + 2x + 2}$.

題號	答案	來源
1	(a)2, (b)1, (c)4, (d)Does not exist, (e)3	1.5 – 習題 5
2	證明題	1.6 – 例題 8*
3	(a) -4 , (b) $\frac{11}{10}$	1.6 – 習題 18
4	證明題	1.6 – 習題 41
5	$c = \frac{2}{3}$	1.8 – 習題 47
6	證明題	1.8 – 習題 55
7	(a)(b)證明題	2.2 – 例題 5
8	$3 - 14x$	2.2 – 習題 22*
9	tangent line is $y = \frac{1}{2}$, normal line is $x = 1$	2.3 – 例題 12*
10	(a) $24x^2 + 40x + 6$, (b) $\frac{x^2 + 2x}{(x^2 + 2x + 2)^2}$	2.3 – 習題 11,31

* 為非勾選習題、勾選習題類似題。

證明題過程略過。