中原大學 114 學年度 ■上學期 □下學期 考試命題紙 ■期中考

科目名稱: 微積分 (上)(A 群)
考試時間: 11 月 5 日第二節

I. 填充題. (25 分)

1. Find $\lim_{x\to 1} \dfrac{\sin(x-1)}{x^2+2x-3} =$ $\dfrac{1}{4}$

2. Find $\lim_{x\to 0} \dfrac{1-\cos 4x}{x^2} =$ $8$

3. Differentiate $f(x) = \sec(\tan(\sin x)), f'(x) =$ $\sec(\tan(\sin x))\tan(\tan(\sin x))\sec^2(\sin x)\cos x$

4. Find an equation of the tangent line to $2x^2+5y^2=7xy$ at $(1,1)$. Ans： $y-1=x-1$

5. Find the linearization of the function $f(x)=\sqrt{x+8}$ at $a=1$. Ans: $y=3+\frac{1}{6}(x-1)$

II. 計算、證明題. (80 分)

1. If we have $\lim_{x\to 0^+} \frac{\sin x}{x} = 1$, use this result to show that $\lim_{x\to 0^-} \frac{\sin x}{x} = 1$

2. Find $\lim_{x\to 0} \tan 4x \cot 3x$ without using L'Hôpital's rule.

3. If $f(x) = \tan^2(\sec^4(x^2+1))$. Find $f'(x)$. (只寫答案須完全正確不然不予計分)

4. If $f(x) = \sin(x + \tan(x + \cos x))$. Find $f'(x)$. (只寫答案須完全正確不然不予計分)

5. Find $y'$ if $\sin(x+y) + x^2y = y^2 \cos x$.

6. If $f(x) = \sin x$, use the definition of derivative to show that $f'(x) = \cos x$.

7. Use implicit differentiation to find an equation of the tangent line to the curve $y \sin 2x = x \cos 2y$ at $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.

8. Let $y = f(x) = \sqrt[3]{x}$. Use the linear approximation of $f$ at $x = 27$ to estimate $\sqrt[3]{26}$.

9. Find the absolute maximum and absolute minimum values of $f(x) = 5 + 54x - 2x^3$ on $[0, 4]$.

10. Find the critical numbers of $f(x) = x^4 - 6x^2 + 1$ and find its local maximum and minimum values using completing square (配方法).

    (局部極值: 使用配方法必須說明何時有局部極值; 直接寫答案或使用其他方法不予計分)

# 114 學年度第 1 學期理、工、電資學院微積分 (A 群) 期中考答案 2025.11.5

| 題號 | 答案 | 來源 |
|---|---|---|
| 1 | 證明題 | 2.4 – 定義 |
| 2 | $\frac{4}{3}$ | 2.4 – 習題 51* |
| 3 | $(6x\tan(\sec^4(x^2+1)))\sec^2(\sec^4(x^2+1))\sec^4(x^2+1)\tan(x^2+1)$ | 2.5 – 習題 35* |
| 4 | $\cos(x+\tan(x+\cos x))(1+\sec^2(x+\cos x)(1-\sin x))$ | 2.5 – 習題 46 |
| 5 | $y'=\frac{\cos(x+y)+2xy+y^2\sin x}{2y\cos x-x^2-\cos(x+y)}$ | 2.6 – 例題 3* |
| 6 | 證明題 | 2.4 – 定義 |
| 7 | Tangent line:$y=\frac{\pi}{4}=\frac{1}{2}(x-\frac{\pi}{2})$ | 2.6 – 習題 25 |
| 8 | $L(26)=3-\frac{1}{27}$ | 2.9 – 習題 34 |
| 9 | Absolute max. is 113, absolute min. is 5 | 3.1 – 習題 50 |
| 10 | critical numbers are $0,\pm\sqrt{3}$, local max. is 1, local min. is -8 | 3.1 – 例題 7,8* |

* 為非勾選習題、勾選習題類似題.
證明題過程略過.