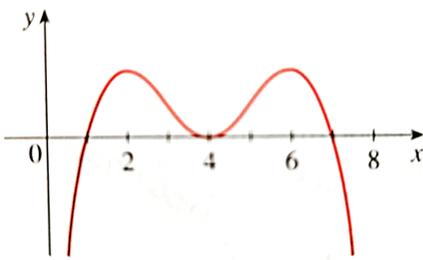科目名稱: 微積分 (上)(A 群)　　　開課班級:A 群微積分　　　考試時間: 12 月 3 日第二節

I. 填充題. (25 分)

1. The function $f(t) = \sin^2(2t)$ satisfies the hypotheses of Rolle's Theorem on the interval $\left[0, \dfrac{\pi}{2}\right]$.

   Find all values of c that satisfy the conclusion of the theorem: $\underline{\dfrac{\pi}{4}}$

2. This is the figure of $f'(x)$. Find the points of inflection: $\underline{x = 2, 4, 6}$

3. Consider $f(x) = x^3 - 3x + 2$ on $[-4, 0]$. Find all possible numbers c that satisfy the conclusion

   of the Mean Value Theorem. Ans: $\underline{-\dfrac{4}{\sqrt{3}}}$

4. Find $\lim\limits_{x \to \infty} 5x - \sqrt{25x^2 + 7} = \underline{0}$

5. Find the slant asymptote of $f(x) = \dfrac{2x^3 - 5x^2 + 3x}{x^2 - x - 2}$. Ans:$\underline{y = 2x - 3}$

II. 計算、證明題. (80 分)

1. Show that the equation $2x + \sin x \cos x = 0$ has exactly one real solution.

2. Suppose f is an odd function and is differentiable everywhere.

   Prove that for every positive number b, there exists a number c in $(-b, b)$ such that $f'(c) = \dfrac{f(b)}{b}$.

3. Sketch the graph of the function $y = f(x)$ that satisfies all of the following conditions.
   $f'(5) = 0, f'(x) > 0$ when $x < 5, f'(x) < 0$ when $x > 5, f''(2) = f''(8) = 0$,
   $f''(x) < 0$ when $x < 2$ or $x > 8, f''(x) > 0$ when $2 < x < 8$.

   Identify the local maximum, local minimum and inflection points if there is any.

4. The graph of the derivative f' of a function f is shown:
   (a) On what open intervals is f increasing? Decreasing?   (b) At what values of x does f have as local maximum? Local minimum?
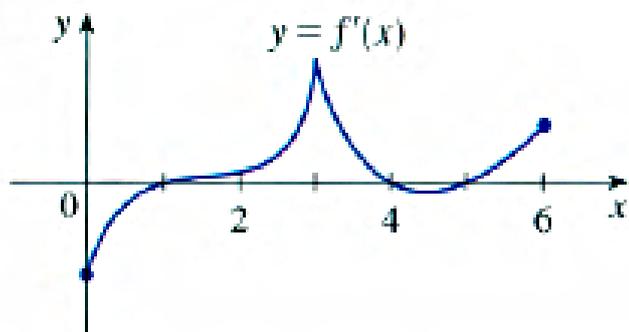
5. Let $f'(x) > 0$ for all $x \in (a, b)$. Show that $f(x)$ is increasing on (a, b) by Mean Value Theorem.

6. Find $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{1 + 4x^6}}{2 - x^3}$.

7. Sketch the curve $f(x) = x^{5/3} - 3x^{2/3}$ by considering increasing/decreasing intervals, concavity, and vertical and slant asymptotes. Locate the local maximum/minimum points, and inflection points if there is any.

8. Sketch the curve $y = \dfrac{x}{\sqrt{x^2 - 1}}$ by considering increasing/decreasing intervals, concavity, and vertical and slant asymptotes. Locate the local maximum/minimum points, and inflection points if there is any.

9. Let A be the area of the region that lies under the graph of $y = 2 - x^2$ for $0 \le x \le 1$.
   (a) Sketch the above region on the xy-plane, and shade the area representing A.
   (b) Divide $[0, 1]$ into 4 equal subintervals. Using the left endpoint of each subinterval as the sample point, sum the areas of the four corresponding rectangles; denote this approximation by L_4. Similarly, using the right endpoint of each subinterval as the sample point, sum the areas of the four corresponding rectangles; denote this approximation by R_4. What is the relationship among A, L_4 and R_4? Explain your reasoning.
   [Hint: (a) 在 xy 平面上畫出區域 A。 (b) 將區間 [0,1] 等分為四段，以四個矩形的面積和估計 A：每矩形若以函數的左 (右) 端點取高，其總和為 L₄(R₄)。請比較 A、L₄、R₄ 的大小並說明原因。]

10. Let A be the area of the region that lies under the graph of $y = 1 + x$ between $x = 0$ and $x = 2$.
    (a) Sketch the above region on the xy-plane, and shade the area representing A.
    (b) Approximate A by dividing $[0, 2]$ into n equal subintervals and summing the areas of the n corresponding rectangles. Use the right endpoint of each subinterval as the sample point to determine the height y of each rectangle. Express your result using Sigma notation.
    (c) Compute your expression in (b) when n approaches infinity for the area, A.

114 學年度第 1 學期理、工、電資學院微積分 (A 群) 第二次會考答案 2025.12.3

題號	答案
1	證明題
2	證明題
3	做圖題
4	$(a) f$ is increasing on $(1,4)$ and $(5,6)$, decreasing on $(0,1)$ and $(4,5)$ $(b) 1$ is a local min. and 4 is a local max.
5	證明題
6	2
7	做圖題
8	做圖題
9	敘述做圖題
10	(a) 做圖,　$(b) \sum_{i=1}^{n}\left(1+\dfrac{2}{n}i\right)\dfrac{2}{n}$

* 為非勾選習題、例題或勾選習題類似題.
證明題、做圖題過程略過.