中原大學 114 學年度 ■上學期 □下學期 考試命題紙 ■期末考

科目名稱: 微積分 (上)(B 群)
考試時間: 1 月 7 日第二節

I. 填充題. (25 分)

1. Find all critical numbers c of $f(x) = x^3 - x + 3$, $c =$ $\pm \frac{1}{\sqrt{3}}$

2. Find the absolute maximum of $f(x) = \frac{x}{x^2+1}$ on $[-2, 4]$. The absolute maximum of $f(x)$ is at $x =$ 1

3. Find a local minimum of $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$. A local minimum of $f(x)$ is at $x =$ 2

4. Let $f(x) = 4x^3 + 12x^2 - 7x$, the inflection point $=$ $(-1, 15)$

5. Let $f(x) = x^4 - 2x^3 - x^2 + 2x$.
   Which of the following statement is true ?
   (a) f is increasing on the interval $(0, 1)$.
   (b) f is decreasing on the interval $(0, 1)$.
   (c) f is concave up on the interval $(0, 1)$.
   (d) f is concave down on the interval $(0, 1)$.

   Ans: d

II. 計算、證明題. (80 分)

1. Let $f(x) = 2x^4 - 8x^3 + 8x^2 - 1$.
   (a) Find the open intervals where the graph of function $f(x)$ is increasing and decreasing.
   (b) Determine all local extrema of the above function.

2. Let $f(x) = x^4 - 6x^2 + 2x + 3$.
   (a) Determine the intervals where the graph of function $f(x)$ is concave up and concave down.
   (b) Identify the inflection points.

3. Find all local extrema of $f(x) = x^{8/3} - 4x^{5/3} + 4x^{2/3}$.

4. Prove that, if $f'(x) > 0$ for all x in an open interval I then f is increasing on I.

5. Find all critical numbers of the function $f(x) = x^5 - 5x^2 + 1$ and use the First Derivative Test to classify each as the location of a local maximum, local minimum or neither.

6. Find all critical numbers of the function $f(x) = x^4 - 18x^2 + 100$ and use the Second Derivative Test to determine all local extrema.

7. Find the absolute extrema of the function $f(x) = 2\cos(x)\sin(x) + \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$.

8. Determine where $f(x) = x^4 - 6x^2 + 1$ is concave up and concave down and also find any inflection points.

9. Determine all significant features and sketch a graph of $f(x) = x|x|$.

10. Let $f(x) = \dfrac{3x^2 - 1}{x}$. Find the slant asymptote and graph the function and its asymptote on the same axes.

114 學年度第 1 學期理、工、電資學院微積分 (B 群) 期末考答案 2026.1.7

題號	答案
1	(a) $f(x)$ are decreasing on$(-\infty,0),(1,2),$ increasing on$(0,1),(2,\infty)$ (b) $f(x)$ is local maximum at$x=1,$ and local minimum are$x=0,2$
2	(a) $f(x)$ concave up on $(-\infty,-1),(1,\infty),$ concave down on $(-1,1)$ (b) inflection points are $(-1,-4),(1,0)$
3	$f(x)$ is local maximum at $x=\frac{1}{2},$ and local minimum are $x=0,2$
4	略
5	$f(x)$ is local maximum at $x=0,$ and local minimum is $x=\sqrt[3]{2}$
6	$f(x)$ is local maximum at $x=0,$ and local minimum are $x=\pm 3$
7	$f(x)$ is absolute maximum at $x=\frac{\pi}{8},$ and absoluate minimum are $x=\frac{\pi}{2}$
8	concave up：$(-\infty,-1),(1,\infty),$ concave down：$(-1,1),$ Inflection points：$(-1,f(-1)),(1,f(1))$
9	略
10	$y=3x$ is the slant asymptote, 做圖 (略)

* 為非勾選習題、勾選習題類似題.
做圖題與證明題過程略過.