

科目名稱: 微積分 (下)(B 群)

考試時間: 6 月 24 日 第二節

I. 填充題. (25 分)

1. The coefficient of x^2 of the Maclaurin series for $f(x) = (1+x)^{\frac{3}{2}}$ is $\frac{3}{8}$

2. If $f(x, y) = x^4 - 3x^2y + y^2$, then $\frac{\partial^2 f}{\partial x \partial y}$ is $-6x$

3. If $f(x, y) = x^2e^{xy} + \sin(\cos y)$, then $f_x = 2xe^{xy} + x^2ye^{xy}$

4. The domain of $f(x, y) = e^{\sqrt{\frac{x}{y}+1}}$ is $\{(x, y) | y \neq 0, \frac{x}{y} + 1 \geq 0\}$

5. If $f(x, y) = xy \cos(2x - y)$, find $\lim_{(x, y) \rightarrow (\pi, \pi)} f(x, y) = -\pi^2$

II. 計算、證明題. (80 分)

1. Find the Maclaurin polynomial $P_2(x)$ of degree 2 for $f(x) = e^{2x} \cos(3x)$.
2. Find the Taylor polynomial $P_3(x)$ of degree 3 for $f(x) = \sin x$ at $x = \frac{\pi}{6}$.
3. Find and sketch the domain for $f(x, y) = \frac{2x}{x^2 - y^2}$.
4. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 + x^3y + y^4}{x^4 - y^4}$ does not exist.
5. Show that $\lim_{(x,y) \rightarrow (1,2)} \frac{(x-1)^2y - 2(x-1)^2}{x^2 - 2x + y^2 - 4y + 5}$ exists.
6. Find $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2(\sin y^2)(\cos z^2)}{x^2 + y^2 + z^2}$.
7. Sketch the graph and several level curves of $f(x, y) = (x-1)^2 + y^2 - 4$.
8. Let $f(x, y) = x^2y - 4x + 3 \sin y$. Find (a) $\frac{\partial^2 f}{\partial x^2}$, (b) $\frac{\partial^2 f}{\partial y^2}$ and (c) $\frac{\partial^2 f}{\partial x \partial y}$.
9. Let $f(x, y) = \int_x^{x+y} \sin t^2 dt$. Find (a) $\frac{\partial f}{\partial x}$ and (b) $\frac{\partial f}{\partial y}$.
10. For $f(x, y) = \frac{1}{x} \cdot \sin(xy^2)$, show that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

題號	答案
1	$P_2(x) = 1 + 2x - \frac{5}{2}x^2$
2	$P_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}(x - \frac{\pi}{6}) - \frac{1}{4}(x - \frac{\pi}{6})^2 - \frac{\sqrt{3}}{12}(x - \frac{\pi}{6})^3$
3	做圖題略
4	證明題略
5	證明題略
6	0
7	做圖題略
8	(a) $\frac{\partial^2 f}{\partial x^2} = 2y$, (b) $\frac{\partial^2 f}{\partial y^2} = -3 \sin y$, (c) $\frac{\partial^2 f}{\partial x \partial y} = 2x$
9	(a) $\frac{\partial f}{\partial x} = -\sin x^2 + \sin(x+y)^2$, (b) $\frac{\partial f}{\partial y} = \sin(x+y)^2$
10	證明題略

* 為非勾選習題、勾選習題類似題。

做圖題與證明題過程略過。