

科目名稱: 微積分(下)(A群)  
考試時間: 3月25日第二節

I. 填充題. (25分)

1. Find  $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\ln x} = \underline{\infty}$

2. Find  $\frac{d}{dx} e^{\pi+x^2} = \underline{e^{\pi+x^2}(2x)}$

3. Find  $\lim_{x \rightarrow \infty} \left(1 + \frac{\pi}{x}\right)^x = \underline{e^\pi}$

4. Find  $\int_e^{e^2} \frac{1}{x \ln x} dx = \underline{\ln 2}$

5. Let  $f(x) = \sqrt{3 - e^x}$ . Then  $f^{-1}(1) = \underline{\ln 2}$

II. 計算、證明題. (80 分)

1. Find  $y'$  if  $e^{x/y} = x - y$ .

2. Evaluate the integral. (a)  $\int x^2 e^x dx$ .      (b)  $\int x \cdot 2^{x^2} dx$ .

3. Evaluate the following limits (a)  $\lim_{x \rightarrow 0^+} \frac{1}{x} e^{-\frac{1}{x}}$ .      (b)  $\lim_{x \rightarrow 0^+} (1 + \sin 4x)^{\cot x}$ .

4. Calculate  $\int_0^1 \tan^{-1} x dx$ .

5. Find  $\int \sec^3 x dx$ .

6. Use logarithmic differentiation to find the derivative of the function  $y = \frac{(x^3 + 1)^4 \sin^2 x}{x^{1/3}}$ .

7. Differentiate  $y = (\cos x + 2)^x$ .

8. Find (a)  $\int \sin 5x \cos 4x dx$ .      (b)  $\int \sin^2 x \cos^5 x dx$ .

9. Evaluate the integral  $\int_0^1 \frac{x^3}{\sqrt{4+x^2}} dx$ .

10. Find (a)  $\int \tan^3 x dx$ .      (b)  $\int \frac{\cos x}{e^x} dx$ .

題號	答案	來源
1	$y' = \frac{y^2 - ye^{x/y}}{y^2 - xe^{x/y}}$	6.3* - 習題 53
2	(a) $x^2e^x - 2xe^x + 2e^x + C$ , (b) $\frac{2x^2}{2\ln 2} + C$	6.4* - 習題 46, 50
3	(a) 0, (b) 4	6.8 - 例題 9*, 10
4	$\frac{\pi}{4} - \frac{1}{2} \ln 2$	7.1 - 例題 5
5	$\frac{1}{2}(\tan x \sec x + \ln  \sec x + \tan x ) + C$	7.2 - 例題 7
6	$y' = \left( \frac{(x^3 + 1)^4 \sin^2 x}{x^{1/3}} \right) \left( \frac{12x^2}{x^3 + 1} + \frac{2 \sin x \cos x}{\sin^2 x} - \frac{1}{3x} \right)$	6.2* - 習題 66*
7	$y' = (\cos x + 2)^x \left( \ln(\cos x + 2) - \frac{x \sin x}{\cos x + 2} \right)$	6.4* - 習題 39*
8	(a) $-\frac{1}{2} \cos x - \frac{1}{18} \cos(9x) + C$ , (b) $\frac{1}{3} \sin^3 x - \frac{2}{5} \sin^5 x + \frac{1}{7} \sin^7 x + C$	7.2 - 例題 9, 2*
9	$\frac{1}{3} \cdot 5^{3/2} - 4 \cdot 5^{1/2} + \frac{16}{3}$	7.1 - 習題 40
10	(a) $\frac{1}{2} \tan^2 x - \ln  \sec x  + C$ , (b) $\frac{1}{2} [-e^{-x} \cos x + e^{-x} \sin x] + C$	7.2 - 例題 7, 8*

\* 為非勾選習題、勾選習題類似題。

證明題過程略過。