

科目名稱: 微積分(下)(A群)

考試時間: 4 月 22 日第二節

I. 填充題. (25 分)

1. Assume $\int \frac{2x+3}{x^2+x-2} dx = \int \frac{A}{x+2} dx + \int \frac{B}{x-1} dx$. Find $(A, B) = \left(\frac{1}{3}, \frac{5}{3} \right)$

2. Find $\int \frac{1}{(1+x^2)^2} dx = \frac{1}{2} \tan^{-1} x + \frac{x}{2(1+x^2)} + C$

3. Find $\int \frac{x+2}{\sqrt{x^2-1}} dx = \sqrt{x^2-1} + 2 \ln |\sqrt{x^2-1} + x| + C$

4. Find $\int_{-\infty}^1 x e^x dx = 0$

5. Find $\int_0^{\infty} \frac{1}{\sqrt{x}(1+x)} dx = \pi$

II. 計算、證明題. (80 分)

1. Find $\int \frac{1}{x^2\sqrt{x^2+1}} dx$.

2. Find $\int \frac{e^x(e^x-1)}{e^{2x}+2e^x+5} dx$. (Hint: Substitute $u = e^x$ or $u = e^x + 1$)

3. Find $\int \frac{4x^2+4x+1}{x(x^2+1)} dx$.

4. Find $\int \frac{x^2+2}{x^3-3x^2+3x-1} dx$.

5. Determine whether the integral is convergent or divergent. Evaluate the integral if it is convergent:

(a) $\int_0^\infty xe^{-x} dx$, (b) $\int_0^3 \frac{1}{x-1} dx$.

6. Find $\int \frac{e^{\tan^{-1}x}}{(1+x^2)^{\frac{3}{2}}} dx$.

7. (a) If $p < 1$, then show $\int_0^1 \frac{1}{x^p} dx$ is convergent.

(b) If $p \geq 1$, then show $\int_0^1 \frac{1}{x^p} dx$ is divergent.

8. Find $\int_0^\infty \frac{1}{e^x+e^{-x}} dx$.

9. Find $\int \frac{x}{\sqrt{3-2x-x^2}} dx$.

10. Sketch the level curve of $g(x, y) = \sqrt{9-x^2-y^2}$ for $k = 0, 1, 2, 3$. (請在 xy 平面等高線圖形上標示出 k 的值)

題號	答案	來源
1	$-\frac{\sqrt{x^2+1}}{x} + C$	7.3 - 例題 3*
2	$\frac{1}{2} \ln((e^x+1)^2+4) - \tan^{-1}\left(\frac{e^x+1}{2}\right) + C$	7.3 - 應用
3	$\ln x + \frac{3}{2} \ln(x^2+1) + 4 \tan^{-1}x + C$	7.4 - 例題 8*
4	$\ln x-1 - 2(x-1)^{-1} - \frac{3}{2}(x-1)^{-2} + C$	7.4 - 例題 4
5	(a)1, (b) $\int_0^3 \frac{1}{x-1} dx$ is divergent.	7.8 - 例題 2, 7
6	$\frac{1}{2} e^{\tan^{-1}x} \left(\frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}} \right) + C$	7.3 - 應用
7	(a)略, (b)略	7.8 - 習題 69
8	$\frac{\pi}{4}$	7.8 - 習題 17*
9	$-\sqrt{3-2x-x^2} - \sin^{-1}\left(\frac{1+x}{2}\right) + C$	7.3 - 例題 7
10	略	14.1 - 例題 11

* 為非勾選習題、勾選習題類似題。
證明題過程、做圖題略過。