中原大學 114 學年度 □上學期 ■下學期 考試命題紙 ■期中考

科目名稱: 微積分 (下)(B 群)
考試時間: 4 月 22 日第二節

I. 填充題. (25 分)

1. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \underline{\quad -\frac{\pi}{3} \quad}$.

2. $\frac{d}{dx}3^{3x^2+1} = \underline{\quad 3^{3x^2+1}\cdot 6x\cdot \ln 3 \quad}$.

3. Let $f(x) = 3x^3 + 2x + 8$. Find $(f^{-1})'(8) = \underline{\quad \frac{1}{2} \quad}$

4. $\frac{d}{dx}\tan^{-1}(3x^2+6x+2) = \underline{\quad \frac{6x+6}{1+(3x^2+6x+2)^2} \quad}$

5. $\frac{d}{dx}\log_2(3x^2+2x+5) = \underline{\quad \frac{6x+2}{\ln 2\cdot(3x^2+2x+5)} \quad}$

II. 計算、證明題. (80 分)

1. Find $\int \frac{2x}{\sqrt{1-x^4}}\,dx.$

2. Find (a) $\frac{d}{dx}e^{\sin x+8x}$. (b) $\int e^{-5x}dx.$

3. Find (a) $\int \frac{e^x}{1+e^x}dx.$ (b) $\int \frac{e^x}{1+e^{2x}}dx.$

4. (a) Find $\sin\left(\tan^{-1}\left(\frac{2}{5}\right)\right)$. (b) Given $\tan^{-1}(2x-3)=\frac{\pi}{4}$, find x.

5. Find $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}dx.$

6. Find $\int \frac{3}{|x|\sqrt{x^6-1}}dx.$

7. Assume that the function $f(x)=\frac{3-x}{x-1}$ for $x\neq 1$ has an inverse is known.
   Find $f^{-1}(x)$ and $(f^{-1})'(1)$.

8. Find $\int x^3e^{x^4}dx$

9. Find $\frac{d}{dx}\left(\sec^{-1}(x^2+2x+5)\right)^3$.

10. Find $\cos\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{1}{5}\right)$. (Hint. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$)

114 學年度第 2 學期理、工、電資學院微積分 (B 群) 期中考答案 2026.4.22

題號	答案	來源
1	$\sin^{-1} x^2 + C$	$6.5 -$ 例題 5
2	$(a)\ e^{\sin x + 8x} \cdot (\cos x + 8), \qquad (b)\ -\frac{1}{5}e^{-5x} + C$	$6.3 -$ 例題 1^*
3	$(a)\ \ln\lvert 1 + e^x\rvert + C, \qquad (b)\ \tan^{-1} e^x + C$	$6.3 -$ 習題 $16, 6.5 -$ 例題 4^*
4	$(a)\ \frac{2}{\sqrt{29}}, \qquad (b)\ x = 2$	$6.4 -$ 例題 6^*
5	$\frac{\pi}{4}$	$6.5 -$ 習題 22^*
6	$\sec^{-1} x^3 + C$	$6.5 -$ 習題 18^*
7	$f^{-1}(x) = \frac{3+x}{x+1}, (f^{-1})'(1) = -\frac{1}{2}$	$6.2 -$ 習題
8	$\frac{1}{4}e^{x^4} + C$	$6.3 -$ 例題 2^*
9	$3\left(\sec^{-1}(x^2+2x+5)\right)^2 \cdot \left(\frac{2x+2}{\lvert x^2+2x+5\rvert\sqrt{(x^2+2x+5)^2-1}}\right)$	$6.5 -$ 例題 1^*
10	$\frac{4-6\sqrt{6}}{25}$	$6.4 -$ 習題

* 為非勾選習題、勾選習題類似題.
證明題過程、做圖題略過.