中原大學 114 學年度第 2 學期 微積分 A 群 第二次會考

I. 填充題. (25 分)

1. Find $\lim\limits_{(x,y)\to(3,2)} \dfrac{x^2y - xy^3}{x - y + 2} =$ $\underline{\quad -2 \quad}$

2. The set of points where $F(x,y) = \dfrac{xy}{e^{x-y} - 1}$ is continuous on $\underline{\quad \{(x,y) : x - y \neq 0\} \quad}$

3. Let $w = (x^2y + xy + 1)^z$ and $x > 0, y > 0, z > 0$.

   Then $\dfrac{\partial w}{\partial z} =$ $\underline{\quad (x^2y + xy + 1)^z \ln(x^2y + xy + 1) \quad}$

4. Let $f(x,y,z) = x^2yz - xyz^3$. Then the gradient of f is

   $\underline{\quad < 2xyz - yz^3, x^2z - xz^3, x^2y - 3xyz^2 > \quad}$

5. The directional derivative of $f(x,y) = 2x^2 + 3y^2$ at $(0,2)$ in the direction from $P(0,2)$ to $Q\left(\dfrac{3}{4}, 3\right)$

   $= \underline{\quad \dfrac{48}{5} \quad}$

II. 計算、證明題. (80 分)

1. Prove that $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^3}{x^2+y^6}$ does not exist.

2. Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{e^{-x^2-y^2}-1}{x^2+y^2}$. (Hint: Use Polar Coordinates to find the limit.)

3. $F(x,y)=\displaystyle\int_y^x \cos(\ln(t))\,dt$. Find $\left.\dfrac{\partial F}{\partial x}\right|_{(1,4)}$ and $\left.\dfrac{\partial F}{\partial y}\right|_{(1,4)}$.

4. Let $e^z = xyz$. Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

5. Let $w=\sqrt{u+v^2}$. Find $\dfrac{\partial^3 w}{\partial u^2 \partial v}$.

6. Let $z=\ln(3x+2y), x=s\sin t, y=t\cos s$. Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.

7. Let $z=x^4+x^2y, x=s+2t-u, y=stu^2$. Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$, $\dfrac{\partial z}{\partial t}$ and $\dfrac{\partial z}{\partial u}$ when $s=4,t=2,u=1$.

8. Let $f(x,y,z)=\dfrac{x}{y+z}$ and let $\boldsymbol{u}^*$ be the unit vector that maximizes the directional derivative $D_{\boldsymbol{u}}f(8,1,3)$. Find $\boldsymbol{u}^*$ and $D_{\boldsymbol{u}^*}f(8,1,3)$.

9. Find equations of the tangent plane and the normal line of the surface $xy^2z^3=8$ at point $(2,2,1)$.

10. Let $F(x,y)=x^4y^2\sin^{-1}\left(\dfrac{y}{x}\right)$. Show that $x\dfrac{\partial F}{\partial x}+y\dfrac{\partial F}{\partial y}=6F$.

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題號	答案	來源
1	略	14.2 − 習題 36
2	-1	14.2 − 習題 53
3	$\left.\dfrac{\partial F}{\partial x}\right\|_{(1,4)} = 1,\ \left.\dfrac{\partial F}{\partial y}\right\|_{(1,4)} = -\cos(\ln 4)$	14.3 − 習題 26
4	$\dfrac{\partial z}{\partial x} = \dfrac{yz}{e^z - xy},\ \dfrac{\partial z}{\partial y} = \dfrac{xz}{e^z - xy}$	14.3 − 習題 44
5	$\dfrac{3}{4}v(u+v^2)^{-5/2}$	14.3 − 習題 63
6	$\dfrac{\partial z}{\partial s} = \dfrac{3}{3s\sin t + 2t\cos s}\sin t + \dfrac{2}{3s\sin t + 2t\cos s}(-t\sin s),$ $\dfrac{\partial z}{\partial t} = \dfrac{3}{3s\sin t + 2t\cos s}s\cos t + \dfrac{2}{3s\sin t + 2t\cos s}\cos s$	14.5 − 習題 13
7	$\dfrac{\partial z}{\partial s} = 1582,\ \dfrac{\partial z}{\partial t} = 3164,\ \dfrac{\partial z}{\partial u} = -700$	14.5 − 習題 25
8	$\boldsymbol{u}^* = \left(\dfrac{1}{3}, \dfrac{-2}{3}, \dfrac{-2}{3}\right), D_{\boldsymbol{u}^*}f(8,1,3) = \dfrac{3}{4}$	14.6 − 習題 31
9	Tangent plane:$x + 2y + 6z = 12$, Normal line:$\dfrac{x-2}{1} = \dfrac{y-2}{2} = \dfrac{z-1}{6}$	14.6 − 習題 49
10	略	14.5 − 習題 57*

* 為非勾選習題、例題或勾選習題類似題.

證明題、做圖題過程略過.