

科目名稱: 微積分 (下)(A 群)

考試時間: 6 月 24 日 第二節

I. 填充題. (25 分)

1. Evaluate $\int_1^2 \int_0^2 (x + y) dy dx = \underline{5}$

2. Find the 3rd Taylor polynomial for $f(x) = \ln x$ at $c = 1$. Ans: $\underline{(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3}$

3. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx = \underline{\frac{\pi}{8}}$

4. Chang the order of integration $\int_0^1 \int_0^x f(x, y) dy dx = \underline{\int_0^1 \int_y^1 f(x, y) dx dy}$

5. Find the absolute maximum value of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y) : 0 \leq x \leq 3, 0 \leq y \leq 2\}$. Ans= 9

II. 計算、證明題. (80 分)

1. Estimate the volume of the solid that lies above the square $R = [0, 2] \times [0, 2]$ and below the elliptic paraboloid $f(x, y) = 16 - x^2 - y^2$. Divide R into 4 equal squares and choose the sample point to be the upper right corner of each squares R_{ij} .
2. Let $f(x, y) = (x - y)(1 - xy)$. Show that $(1, 1)$ is a saddle point.
3. Let x, y and z be positive numbers.
 - (a) If $x + y + z = 12$, find the minimum value of $x^2 + y^2 + z^2$.
 - (b) Then use the second derivative test to verify your answer in (a).
4. Evaluate $\iint_R y \cos(xy) dA$, where $R = [0, 1] \times [0, \pi]$.
5. Find the volume of the solid that lies under the plane $z = x$ and above the region D in the xy -plane bounded by the line $y = 2x$ and the parabola $y = x^2$.
6. Evaluate the area bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.
7. Find the Maclaurin series for (a) $\frac{1}{\sqrt[3]{1+x}}$ and (b) $\tan^{-1} x$.
8. (a) Find the 9th Maclaurin polynomial for e^{-x^3} .
(b) Use (a) to approximate $\int_0^1 e^{-x^3} dx$.
9. Evaluate $\iint_R (3x + 1) dA$, where R is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
10. Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2} dy dx$ by converting to polar coordinates.

題號	答案
1	44
2	略
3	(a) The minimum value is 48, (b) 略
4	2
5	$\frac{4}{3}$
6	18
7	(a) $\frac{1}{\sqrt[3]{1+x}} = 1 - \frac{x}{3} + \frac{2}{9}x^2 - \dots$, (b) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
8	(a) $e^{-x^3} = 1 - x^3 + \frac{x^6}{2} - \frac{x^9}{6}$, (b) $\int_0^1 e^{-x^3} dx = \frac{169}{210}$
9	$\frac{3}{2}\pi$
10	$\frac{\pi}{4}(1 - e^{-4})$

* 為非勾選習題、勾選習題類似題。
 做圖題與證明題過程略過。